

New Developments in Decision Theory under Risk

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Motivation

- Expected utility (EU) is generally acknowledged as the rational choice criterion under risk
- So also R&D decisions should be made according to EU
- However, in many empirical studies violations of EU have been observed
- Not only single students, but also groups, people holding a PhD, and professionals
- This evidence has motivated the development of alternative theories

Motivation

- Weighted Utility
- Semi-Weighted Utility
- Implicit Expected Utility
- Regret Theory
- Disappointment Theory
- Anticipated Utility
- Dual Expected Utility
- Prospect Theory
- Cumulative Prospect Theory
- Prospective Reference Theory
- Local Expected Utility
- Configural Weight Theory
- Expected Cardinality-Specific Utility
- Theory of the Gambling Effect
- Theory of Pessimism
- SSB Theory
- Rank- and Sign-Dependent Utility
- Mixed-Fan Utility
- Utility with Probability-Dependent Outcomes

Motivation

- Evidence and alternative theories can be useful for
 - Descriptive models of R&D investments
 - Decision support (detection of potential biases)
- Although prospect theory is rather successful, several phenomena remain unexplained
- Recent models with state-dependent reference points increase descriptive power substantially

Outline

- EU and Prospect Theory
- Empirical Evidence
- Models with State-Dependent Reference Points
- Applications

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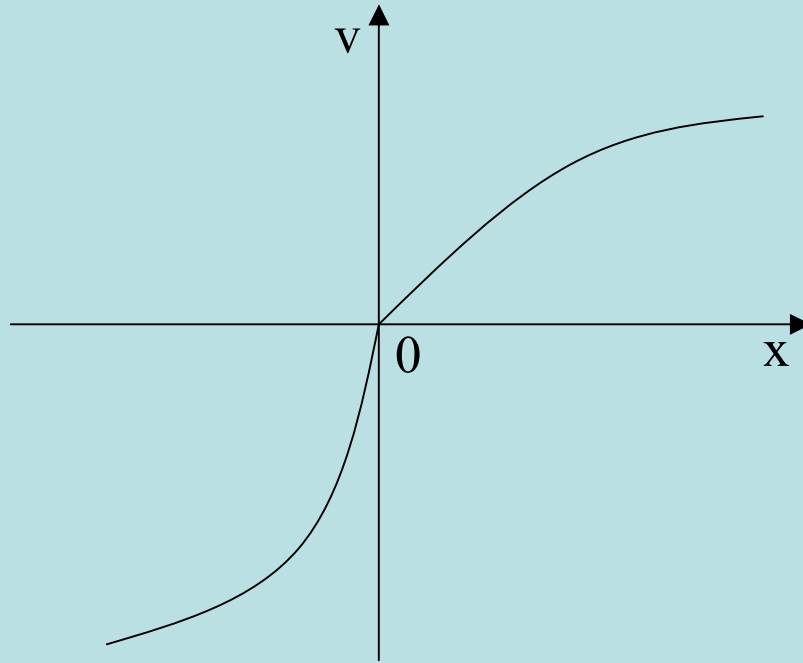
Notation

- S : finite state space (states s_i , $i = 1, \dots, n$)
- p_i : probability of state s_i
- X : set of consequences (interval of the real line)
- F : set of all lotteries
- $f \in F$: function from S to X , i.e. a lottery f specifies for each state s_i the resulting consequence $f_i \in X$
- h : reference lottery

EU and Prospect Theory

- EU: $V(f) = \sum_i u(f_i)p_i$
- Prospect Theory: $V(f, h) = \sum_i v(f_i-h)\pi_i$
 - π_i : decision weight of state i
 - Original prospect theory: $\pi_i = w(p_i)$
 - Utility is defined on gains and losses relative to a reference point h
 - No clear hypothesis on the location of the reference point
 - Usually initial wealth
 - h is deterministic

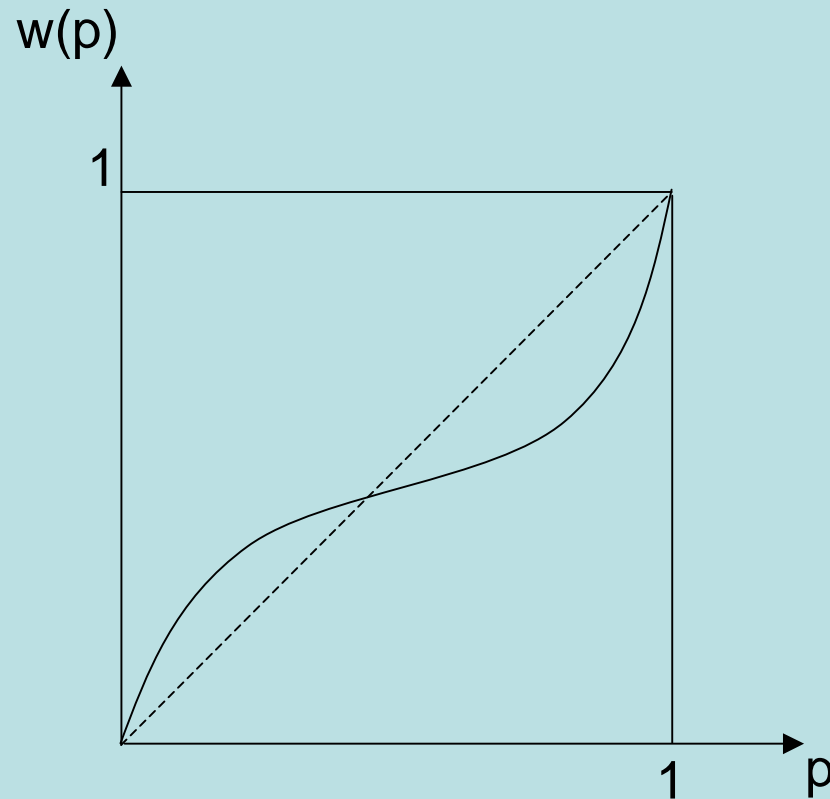
Prospect Theory



Diminishing Sensitivity

Loss Aversion

Prospect Theory



Empirical Evidence: Inverse-S

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Allais Paradox

$$0,25u(3000) < 0,2u(4000)$$

Allais Paradox

Allais Paradox can be explained by Prospect Theory due to probability weighting

Disparity between WTA and WTP

- WTA: $V(W + f) = V(W + WTA)$
- WTP: $V(W) = V(W + f - WTP)$
- EU: $WTA \approx WTP$
- Experiments: $WTA \approx 2,5 * WTP$
- Not explained by Prospect Theory

Preference Reversal

- S = safe lottery
- R = risky lottery
- EU:
 - $S \succ R$ and $WTA_S > WTA_R$
 - $R \succ S$ and $WTA_R > WTA_S$
- Experiments:
 - Standard PR: $S \succ R$ and $WTA_S < WTA_R$
 - Non-standard PR: $R \succ S$ and $WTA_R < WTA_S$

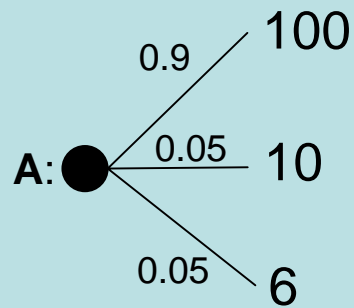
Preference Reversal

- Standard PR occur much more frequently than non-standard ones

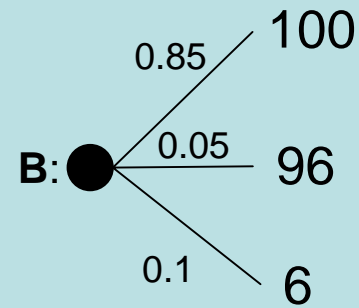
Choice	Consistent	Inconsistent
S: 76	51%	49%
R: 107	84%	16%

- Not explained by Prospect Theory

Event Splitting Effects



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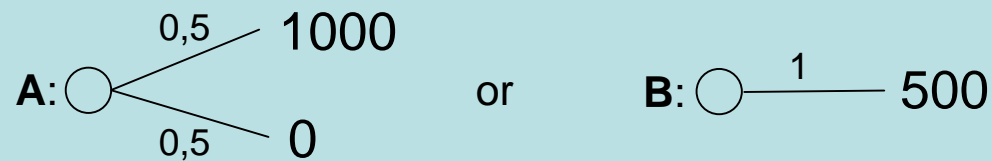
Reference Dependence

- First proposed by Markowitz (1952)
- Core concept of Prospect Theory
- Has received support in several empirical studies
- Example: Isolation Effect

Isolation Effect

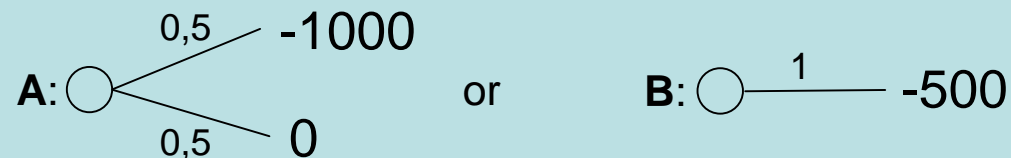
Pair 1:

You get a sure payment of 1000. Additionally you have the choice between:



Pair 2:

You get a sure payment of 2000. Additionally you have the choice between:



Example of Rabin (2000)

- Coin flip with heads = +11€ and tails = -10€
- Coin flip is rejected independently of initial wealth and EU holds
- How big must x be such that a coin flip with heads = + x € and tails = -100€ is accepted?
- There is no x !
- Rabin: Reasonable degrees of risk aversion for small stakes imply unrealistic high degrees of risk aversion for large stakes
- Problem can only be avoided if utility depends on gains and losses and not on final wealth

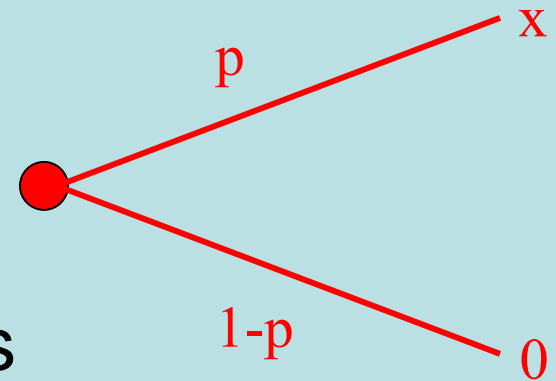
- Although reference dependence and loss aversion are plausible, their descriptive power is limited
- In choice under certainty loss aversion can explain the WTP-WTA disparity if there is a separate reference point for each good
- Idea: there is a reference point for each state of the world

State-Dependent Reference Points

- Expected Utility with State-Dependent Reference Points [Sugden (2003)]:
$$V(f, h) = \sum_i v(f_i - h_i) p_i$$
- Can explain disparity between WTP and WTA as well as preference reversals

State-Dependent Reference Points

- Consider the lottery (x, p)
- Suppose the reference lottery is given by (stochastic) initial wealth
- $$v(x) = \begin{cases} x & \text{if } x > 0 \\ \lambda x & \text{if } x < 0 \end{cases}$$
- WTP: $0 = p(x - \text{WTP}) + (1-p)\lambda(0 - \text{WTP})$
- $\text{WTP} = px / (p + (1-p)\lambda)$



State-Dependent Reference Points

- WTA: $0 = p\lambda(WTA-x) + (1-p)(WTA-0)$
- $WTA = \lambda px / (p\lambda + (1-p))$
- $WTA > WTP$ iff $\lambda > 1$ (loss aversion)
- Also preference reversals if $\lambda > 1$

State-Dependent Reference Points

- Sugden's model cannot explain the Allais Paradox
- Modified Prospect Theory (MPT)
(Schmidt, Starmer, and Sugden 2006):
$$V(f, h) = \sum_i v(f_i - h_i) \pi_i$$
- Special cases:
 - Sugden's model if $\pi_i = p_i$
 - Prospect Theory if $h_i = h_j$ for all i, j

MPT and Preference Reversal

- Preference reversals can be explained in MPT by loss aversion and/or probability weighting
- Parametric analysis:
 - Is explanation of preference reversals by state-dependent reference points plausible?
 - Is loss aversion or probability weighting more important

Parametric Analysis

- Functional forms:

- $v(x) = \begin{cases} x^a & \text{if } x > 0 \\ -\lambda |x|^a & \text{if } x < 0 \end{cases}$

- $w(p) = p^c / (p^c + (1-p)^c)^{1/c}$ [Tversky and Kahneman (1992), Prelec (1998)]

- Plausible values: $0 < a < 1$, $\lambda > 1$, and $0.4 < c < 1$

Lottery Pairs

S-R pair	x	p	y	q
Pair 1	1	0.9	9	0.1
Pair 2	1	0.9	4.5	0.2
Pair 3	1	0.9	3	0.3
Pair 4	1	0.9	2	0.45
Pair 5	1	0.9	1.5	0.6
Pair 6	1.5	0.6	9	0.1
Pair 7	2	0.45	9	0.1
Pair 8	4.5	0.2	9	0.1
Pair 9	1	0.8	4	0.2

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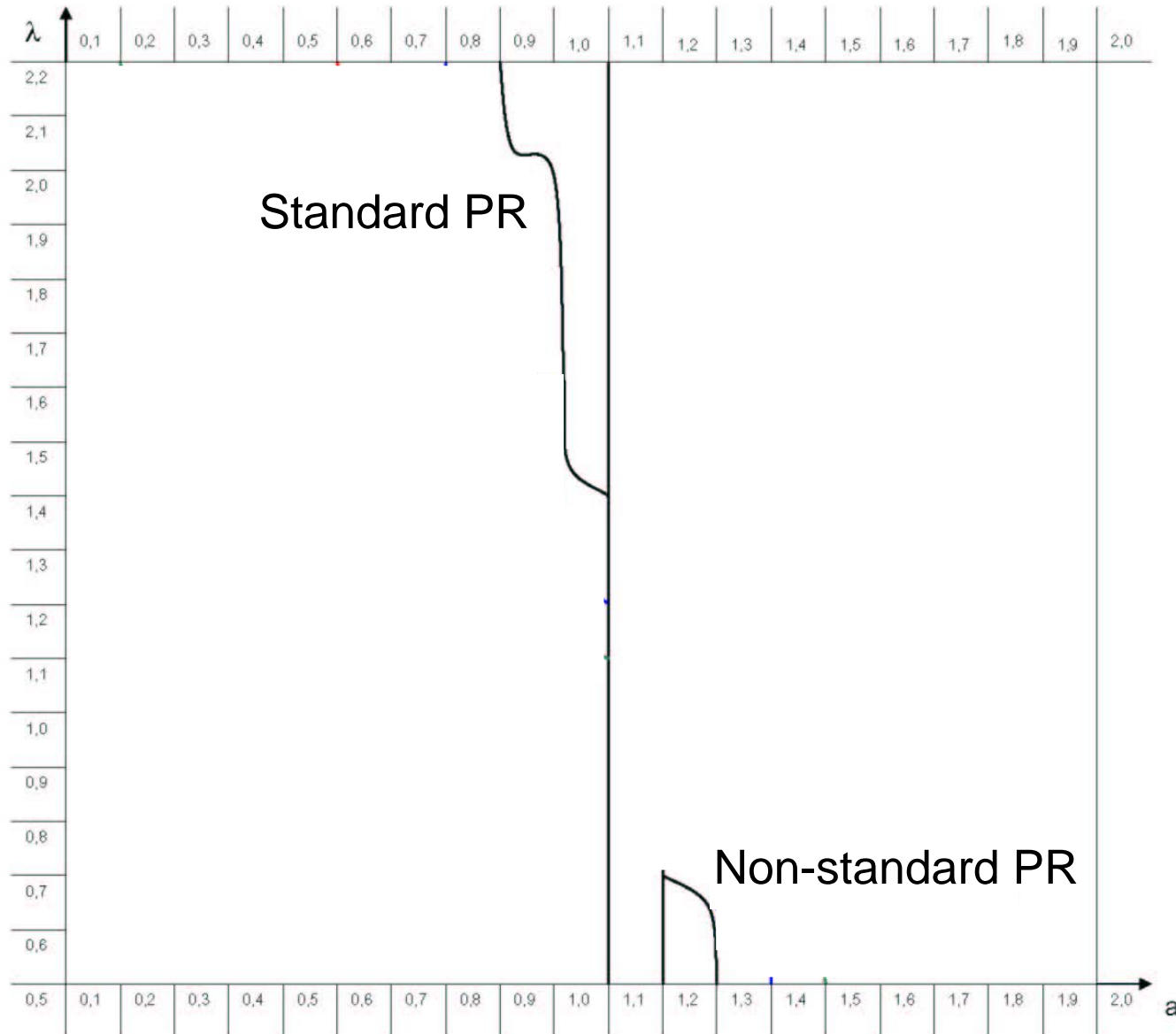
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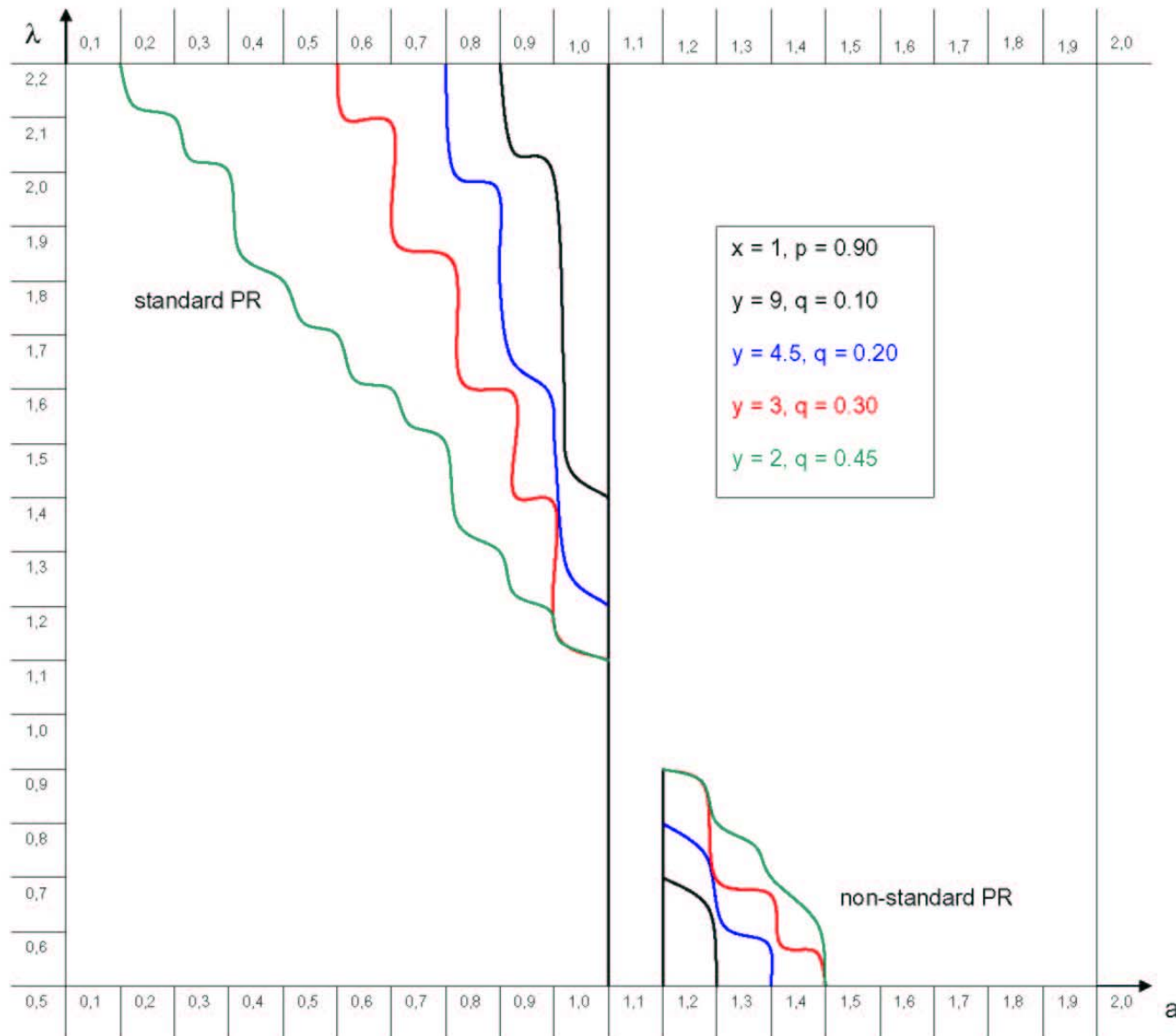
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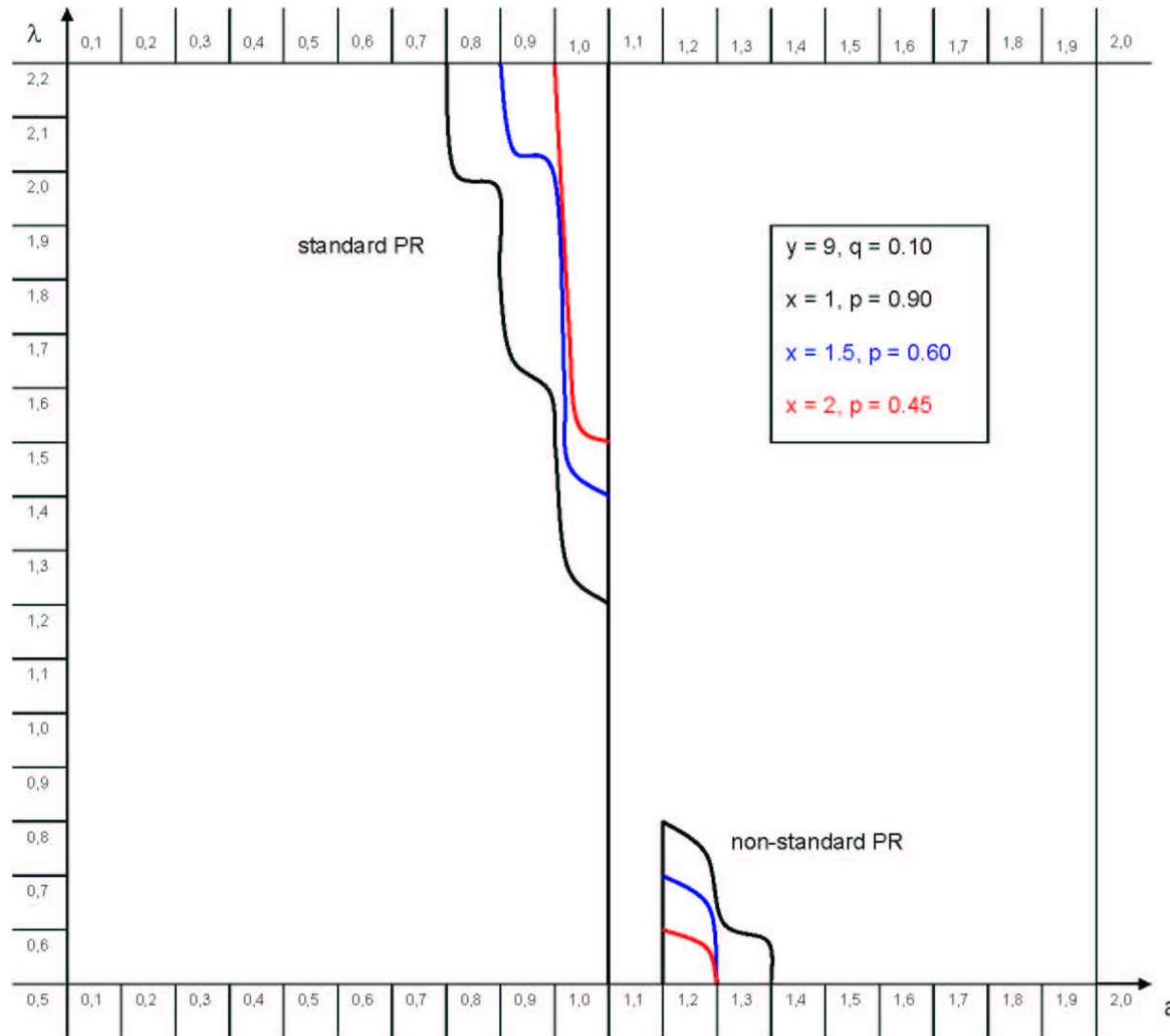
PR and Loss Aversion ($c = 1$), Pair 1



PR and Loss Aversion ($c = 1$), Pairs 1-4



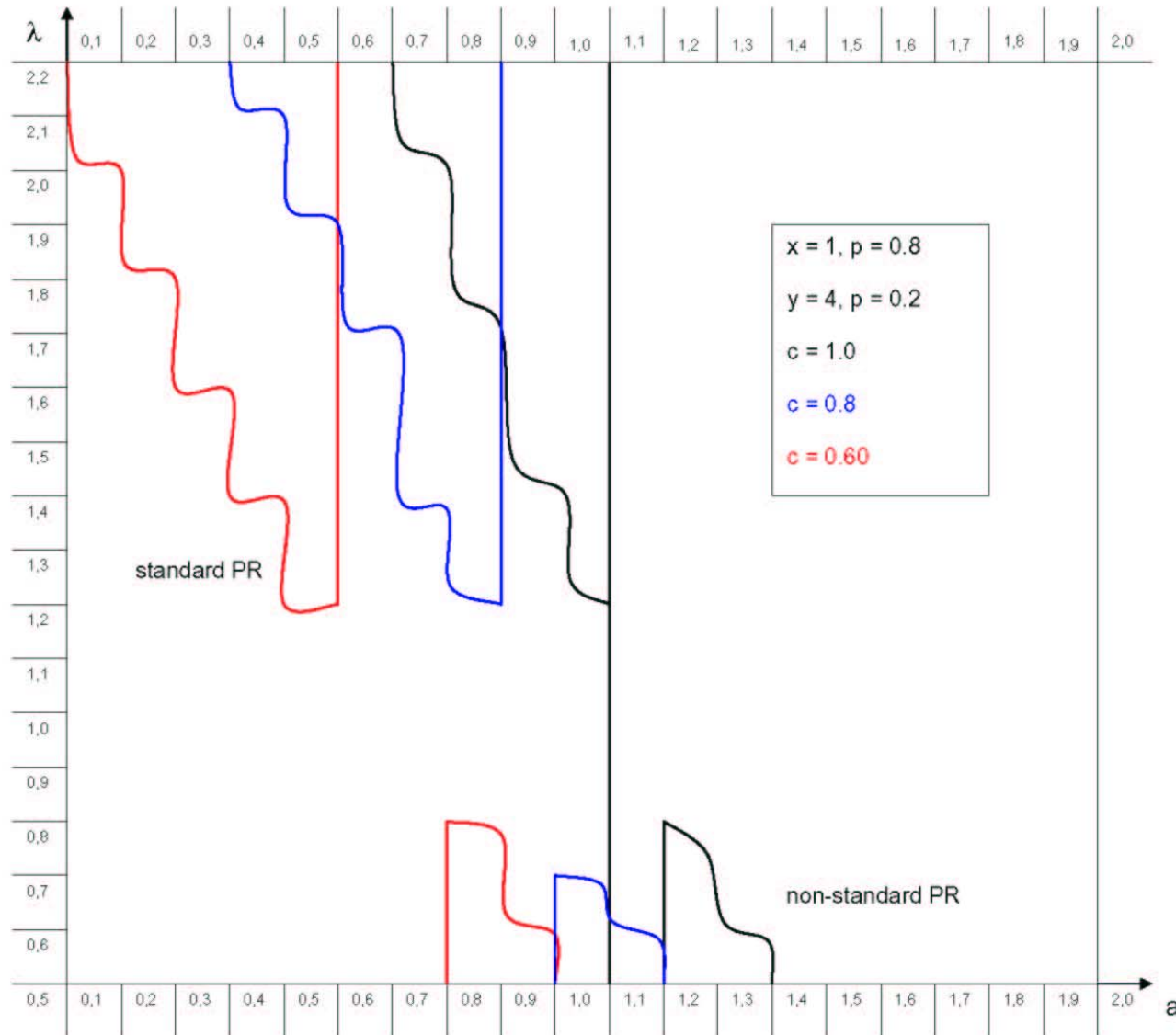
Increasing x ($c = 1$), Pairs 1, 6, 7



PR and Probability Weighting ($\lambda = 1$)

- Pair 8: only non-standard PR
 - Pair 9: only standard PR
 - Pair 5: neither standard nor non-standard PR
- ⇒ Explanation of PR by probability weighting is very sensitive to the parameters of the involved gambles, at least if we have identical weighting functions of gains and losses

Loss Aversion and Probability weighting, Pair 9



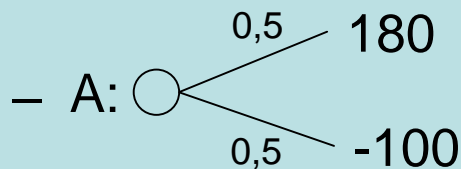
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Applications of Reference Dependence

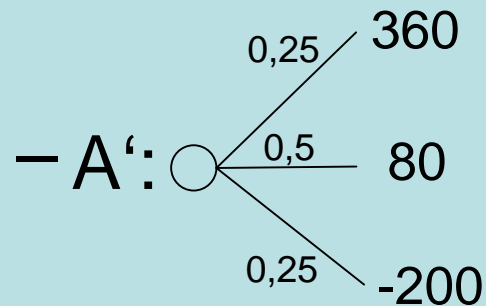
- Equity Premium Puzzle [Mehra & Prescott (1985)]
- Myopic Loss Aversion [Bernartzi & Thaler (1995)]:
 - Loss Aversion
 - Mental Accounting (Short Evaluation Periods)
- Example:

$$- v(x) = \begin{cases} x & \text{if } x > 0 \\ 2x & \text{if } x < 0 \end{cases}$$



Applications of Reference Dependence

- Lottery A played two times in row:



- A' will be accepted
- Separate Evaluation of R&D projects implies in the case of loss aversion too low investments